# SILIGURI INSTITUTE OF TECHNOLOGY
## DEPARTMENT OF MASTER OF BUSINESS ADMINISTRATION (MBA)
## Assignment
(For Advanced Learners)
**MBA 1ST SEMESTER'21**

PAPER**: Quantitative Techniques** CODE: **MB 106**

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**MCQ/Very Short Answer Type** [CO 1]

1. For maximization in Transportation Problem, the objective is to maximize the total ----------
   a) Solution b) Profit Matrix c) Profit d) None of the above
2. If the total supply is less than the total demand, a dummy source (row) is included in the cost matrix with ------------
   a) Dummy Demand b) Dummy Supply c) Zero Cost d) Both A and B
3. -------------- are expressed is n the form of inequities or equations
   a) Constraints b) Objective Functions c) Both A and B d) None of the above
4. The objective functions and constraints are linear relationship between -------------
   a) Variables b) Constraints c) Functions d) All of the above
5. Graphic method can be applied to solve a LPP when there are only ------------ variable
   a) One b) More than One c) Two d) Three
6. If the feasible region of a LPP is empty, the solution is --------------------
   a) Infeasible b) Unbounded c) Alternative d) None of the above
7. The variables whose coefficient vectors are unit vectors are called ------------
   a) Unit Variables b) Basic Variables c) Non basic Variables d) None of the above
8. Any column or raw of a simplex table is called a --------------
   a) Vector b) Key column c) Key Raw d) None of the above
9. If there are 'm' original variables and 'n' introduced variables, then there will be ---------- columns in the simplex table
   a) m + n b) m – n c) 3 +m + n d) m + n – 1
10. A minimization problem can be converted into a maximization problem by changing the sign of coefficients in the
    a) Constraints b) Objective Functions c) Both A and B d) None of the above
11. If in a LPP, the solution of a variable can be made infinity large without violating the constraint the solution is
    a) Infeasible b) Unbounded c) Alternative d) None of the above
12. In maximization cases, ---------- are assigned to the artificial variables as their coefficients in the objective function
    a) +m b) –m c) 0 d) None of the above
13. In simplex method, we add ---------------- variables in the case of '='
    a) Slack Variable b) Surplus Variable c) Artificial Variable d) None of the above
14. In simplex method, if there is tie between a decision variable and slack (or surplus) variable,---- should be selected
    a) Slack variable b) Surplus variable c) Decision variable d) None of the above
15. A BFS of a LPP is said to be ---------------- if at least one of the basic variable is zero
    a) Degenerate b) Non-degenerate c) Infeasible d) Unbounded
16. In LPP, degeneracy occurs in --------------- stages
    a) One b) Two c) Three d) Four
17. Every LPP is associated with another LPP is called -----------------
    a) Primal b) Dual c) Non-linear programming d) None of the above
18. As for maximization in assignment problem, the objective is to maximize the -----------
    a) Profit b) Optimization c) Cost d) None of the above
19. If there is more than one optimum solution for the decision variable the solution is -------------
    a) Infeasible b) Unbounded c) Alternative d) None of the above
20. Dual of the dual is --------------
    a) Primal b) Dual c) Alternative d) None of the above
21. The term linearity implies ----------- among the relevant variables:
    a) Straight line b) Proportional relationships c) Linear lines d) Both A and B
22. The world 'programming' means taking decisions -------------
    a) Systematically b) Rapidly c) Slowly d) Instantly
23. LP model is based on the assumptions of -----------------
    a) Proportionality b) Additivity c) Certainty d) All of the above

24. ------------- assumption means the prior knowledge of all the coefficients in the objective function, the Coefficients of the constraints and the resource values.

a) Proportionality b) Certainty c) Finite choices d) Continuity

25. Simple linear programming problem with ---------------- variables can be easily solved by the graphical method.

a) One decision b) Four decisions c) Three decisions d) Two decisions

26. Any solution to a LPP which satisfies the non- negativity restrictions of the LPP is called its --------

a) Unbounded solution b) Optimal solution c) Feasible solution d) Both A and B

27. Any feasible solution which optimizes (minimizes or maximizes) the objective function of the LPP is called its

a) Optimal solution b) Non-basic variables c) Solution d) Basic feasible solution

28. What is also defined as the non-negative variables which are added in the LHS of the constraint to convert the inequality '< 'into an equation?

a) Slack variables b) Simplex algorithm c) Key element d) None of the above

29. Which method is an iterative procedure for solving LPP in a finite number of steps?

a) Simplex algorithm b) Slack variable c) M method d) Simplex method

30. In simplex algorithm, which method is used to deal with the situation where an infeasible starting basic solution is given?

a) Slack variable b) Simplex method c) M- method d) None of the above

31. Consider a random experiment of throwing a die. What is the probability of getting odd face?

(a) 1/6 (b) 2/3 (c) 1/2 (d) 0

32. Consider the random experiment of choosing a card. What is the probability of getting queen?

(a) 1/52 (b) 1/13 (c) 2/13 (d) 1

33. If A and B are independent event, then P(AB)=

a) P(A) b) P(B) c) P(A)P(B) d) 0

34. If Cov(x, y) = 12, $\sigma_x = 5$ and $r_{xy} = 0.6$ then $\sigma_y =$

a) 16 b) 8 c) 2 d) 4

34. The correlation coefficient between two variables x and y is

(a) $\frac{cov(x,y)}{\sigma_x \sigma_y}$ (b) $\frac{cov(x,y)}{\sigma_{x+}\sigma_y}$ (c) $\frac{cov(x,y)}{\sigma_{x-}\sigma_y}$ (d) 0

## **Short Answer Type** (5 Marks):

1. Find the Optimal Assignment schedule of following machine & job allocation problem

| | J1 | J2 | J3 | J4 | J5 |
|---|---|---|---|---|---|
| M1 | 9 | 11 | 15 | 10 | 11 |
| M2 | 12 | 9 | -- | 10 | 9 |
| M3 | -- | 11 | 14 | 11 | 7 |
| M4 | 14 | 8 | 12 | 7 | 8 |

2. Find the Dual of the following LPP:

Maximize $Z = 4x_1 + x_2 + 7x_3$

Subject to Constraints: $x_1 + 7x_2 - 3x_3 \le 4;$

$5x_1 - x_2 + x_3 \ge 12;$

$x_1 + x_2 + x_3 = 10$

Where all the $x_1, x_2, x_3 \ge 0$

3. Apply the Principal of Dominance to solve the following game whose pay-offs are given below:-

$$\begin{pmatrix} 4 & 7 & 1 \\ 3 & 6 & -4 \\ -2 & -1 & 2 \end{pmatrix}$$

4. Solve the following LPP by Graphical method

Maximize $Z = 40x_1 + 60x_2$

Subject to Constraints: $2x_1 + x_2 \le 70;$ $x_1 + x_2 \ge 40;$ $x_1 + 3x_2 \le 90$ **&** Where all $x_1, x_2 \ge 0$

5. Find the Dual of the following LPP

Maximize $Z = 4x_1 + x_2 + 7x_3$

Subject to Constraints: $x_1 + 7x_2 - 3x_3 \le 4;$ $5x_1 - x_2 + x_3 \ge 12;$ $x_1 + x_2 + x_3 = 10$ **&** Where all the $x_1, x_2, x_3 \ge 0$

6. The frequency distribution of the marks obtained by 90 students is given below. Compute the mean and standard deviation.

| **Marks** | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 |
|---|---|---|---|---|---|---|---|---|
| **No. Of Students** | 5 | 12 | 15 | 20 | 18 | 10 | 6 | 4 |

7. Write the definition probability? An integer is chosen at random from the first 100 positive integers. What is the probability that the integer is divisible by 6 or 8?

8. Three coins are throwing. What is the probability of not getting three tails? One die is throwing. What is the probability of getting the number divisible by 3?

(i) If X is normally distributed with mean 11 and standard deviation 1.5, then find the probability of x = 1

(ii) Consider the random experiment of tossing a fair coin till a head appears for the first time. Let X is the number of tosses required. Find the distribution of X.

(iii) A system that will either operate or fail in a certain event mission and let p denotes the probability of the successful operation. Eight trails are considered with the result S, F, S, S, S, F, S, S. Assuming independence of the trails find the maximum likelihood estimates of p.

## **Long Answer Type** (15 Marks)

1. Find the Initial Basic Feasible Solution by VAM & Check Optimality of the following Transportation Problem using MODI method (CO3)

| | W1 | W2 | W3 | W4 | W4 | ***Supply*** |
|---|---|---|---|---|---|---|
| F1 | 55 | 30 | 40 | 50 | 50 | 40 |
| F2 | 35 | 30 | 100 | 45 | 60 | 20 |
| F3 | 40 | 60 | 95 | 35 | 30 | 40 |
| ***Demand*** | 25 | 10 | 20 | 30 | 15 | |

2. Find the Initial Basic Feasible Solution by VAM of the following Transportation Problem. (CO3)

| | W1 | W2 | W3 | W4 | W4 | ***Supply*** |
|---|---|---|---|---|---|---|
| F1 | 55 | 30 | 40 | 50 | 50 | 40 |
| F2 | 35 | 30 | 100 | 45 | 60 | 20 |
| F3 | 40 | 60 | 95 | 35 | 30 | 40 |
| ***Demand*** | 25 | 10 | 20 | 30 | 15 | |

3. (a) State Bayes' Theorem and write the mathematical expression of the Theorem.

(b) From a box containing 5 white balls and 5 black balls, 5 balls are transferred at random into an empty second box from which one ball is drawn and it is found to be white. What is the probability that all balls transferred from the first box are white? (3+7)

4. (a) Define correlation coefficient and show that the value of correlation coefficient lies between -1 and +1

(b) Find the correlation coefficient of the following data

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| x | 65 | 63 | 67 | 64 | 68 | 62 | 70 | 66 |
| y | 68 | 66 | 68 | 65 | 69 | 66 | 68 | 65 |

5. Solve by Big – M method

Minimize $Z = 5x_1 + 3x_2$

Subject to Constraints: $2x_1 + 3x_2 \leq 18;$

$2x_1 + 5x_2 \leq 10;$

$3x_1 + 5x_2 \geq 15$

**&** Where all $x_1, x_2 \geq 0$

6. The following table gives the ages and blood pressure of 10 women

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Age(X) | 56 | 42 | 36 | 47 | 49 | 42 | 60 | 72 | 63 | 55 |
| Blood Pressure(Y) | 147 | 125 | 118 | 128 | 145 | 140 | 155 | 160 | 149 | 150 |

Determine the (a) regression line of Y on X (b) regression line of X on Y (c) correlation coefficient between X & Y and (d) Estimate the blood pressure of women whose age is 45 years.

## 10.4 The Simplex Method for Maximization Problems

The simplex method is a computational procedure - *an algorithm* - for solving linear programming problems. It is an iterative optimizing technique. In the *simplex process, we must first find an initial basis solution (extreme point). We then proceed to an adjacent extreme point. We continue moving from point to point until we reach an optimal solution.* For a maximization problem, the simplex method always moves in the direction of steepest ascent thus ensuring that the *value of the objective function improves with each solution.*

**Simplex Algoritham**

*Maximize*

$Z = c_1 x_1 + c_2 x_2 + \ldots\ldots\ldots\ldots + c_n x_n$

*Subject to the constraints*

$k_{11}x_1 + k_{12}x_2 + \ldots\ldots + k_{1n}x_n \leq b_1$

$k_{21}x_1 + k_{22}x_2 + \ldots\ldots + k_{2n}x_n \leq b_2$

$k_{m1}x_1 + k_{m2}x_2 + \ldots\ldots + k_{mn}x_n \leq b_m$

$x_1, x_2, \ldots x_n \geq 0$

- $x_1, x_2 \ldots\ldots x_n$ are the **'Decision Variables'**.
- $c_j$ (j = 1, 2,......., n) in the objective function are called the **'Profit or Cost coefficients'**.
- $b_i$ (i = 1, 2 .......m) are called **'Resources'**.
- Constants $k_{ij}$ (i = 1,2,......,m; j = 1, 2,.....n) are called **'Structural Coefficients'**.

- **An inequality of the "≤" type is changed into an equality:**
  By the addition of a non-negative slack variable. By adding a suitable positive quantity $s_i$ to the left hand side, the inequality constraint can be written as:
  $k_{i1}x_1 + k_{i2}x_2 + \ldots\ldots + k_{in}x_n + s_i = b_i$ (i = 1, 2, .....m)
- **An inequality of the "≥" type is changed into an equality:**
  By the subtraction of a non negative surplus variable. By subtracting positive quantity $s_i$ from the left hand side, the inequality constraint can, be written as:
  $k_{i1}x_1 + k_{i2}x_2 + \ldots\ldots + k_{in}x_n - s_i = b_i$ (i = 1, 2, .....m)

**Solution Steps:**

$Z = c_1 x_1 + c_2 x_2 + \ldots\ldots\ldots\ldots c_n x_n + 0s_1 + 0s_2 + \ldots\ldots\ldots + 0s_m$ (Objective function)

*Subject to the constraints*

$k_{11}x_1 + k_{12}x_2 + \ldots\ldots + k_{1n}x_n + s_1 = b_1$

$k_{21}x_1 + k_{22}x_2 + \ldots\ldots + k_{2n}x_n + s_2 = b_2$

$k_{m1}x_1 + k_{m2}x_2 + \ldots\ldots + k_{mn}x_n + s_m = b_m$

$x_1, x_2, \ldots x_n, s_1, s_2, \ldots s_m \geq 0$

- Note that *slack variables have been allocated zero coefficients in the objective function* as these variables typically contribute nil amount to the value of objective function.

**INITIAL SIMPLEX TABLEAU**

| **$C_j$ (Contribution per Unit)** | | | **$c_1$** | **$c_2$** | **.....** | **$c_n$** | **0** | **0** | **.....** | **0** | **Minimum Ratio $X_B / k_{ij}$** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **$C_B$** | **Basic Variables (B)** | **Value of Basic Variables b=($X_B$)** | **$x_1$** | **$x_2$** | **.....** | **$x_n$** | **$s_1$** | **$s_2$** | **.....** | **$s_m$** | |
| | | | **Coefficient Matrix** | | | | **Identity Matrix** | | | | |
| $C_{B_1}$ | $s_1$ | $b_1 = x_{B_1}$ | $k_{11}$ | $k_{12}$ | ..... | $k_{1n}$ | 1 | 0 | ..... | 0 | |
| $C_{B_2}$ | $s_2$ | $b_2 = x_{B_2}$ | $k_{21}$ | $k_{22}$ | ..... | $k_{2n}$ | 0 | 1 | ..... | 0 | |
| - | - | - | - | - | ..... | - | - | - | ..... | - | |
| - | - | - | - | - | ..... | - | - | - | ..... | - | |
| $C_{B_m}$ | $s_m$ | $b_m = x_{B_m}$ | $k_{m1}$ | $k_{m2}$ | ..... | $k_{mn}$ | 0 | 0 | ..... | 1 | |
| $Z_j = \sum C_{Bi}X_j$ | | | 0 | 0 | ..... | 0 | 0 | 0 | ..... | 0 | |
| Net Contribution *per unit* ($C_j - Z_j$) | | | $c_1 - z_1$ | $c_2 - z_2$ | ..... | $c_n - z_n$ | 0 | 0 | ..... | 0 | |

- **In the first row '$C_j$'** we write the coefficients of the variables in the objective function ($c_1$, $c_2$, $c_3$ .......,$c_n$, 0,0.....0). These value will remain unchanged in the subsequent tableaus.
- **In the first column labeled '$C_B$'**, we list the coefficients of the current basic variables in the objective function.
- **In the second column 'Basic Variables'** we place the basic variables ($s_1$, $s_2$, $s_3$ .....$s_m$)
- **'The Coefficient Matrix' (under $x_1$,$x_2$......$x_n$)** in the tableau represents the coefficients of the decision variables in the constraints set.
- **'The Identity Matrix' (under $s_1$, $s_2$.......$s_m$)** in the initial simplex tableau represents the coefficients of the slack variables in the constraint set.
- **To get $Z_j$ row under a column**, we multiply the entries of that column by the corresponding entries of $C_B$ column and add the product.
- The last row '$C_j – Z_j$' called the 'Index Row or Net Evaluation Row'.
- *If all the elements or entries in the $C_j – Z_j$ row are Negative or Zero then the* **current solution is optimum**.
- **If current solution is not optimum**, it can be further enhanced by eliminating one basic variable and replacing it by some non-basic one. For this
  - ✓ We now decide the **variable to enter** the solution mix. *Column with largest positive entry in the $C_j – Z_j$ row is called* **'key Column'** *(indicated by ↑)*. The non-basic variable at the top of the column is the entering variable that will replace a basic variable.
  - ✓ Next step is to **determine 'leaving variable'** to be replaced. This can be trace by dividing each number in the $X_B$ column by the corresponding number in key column. We compute ratio ($b_1/k_{1j}$, $b_2/k_{2j}$ ..... $b_m/k_{mj}$). The row corresponding to the minimum of these ratios is key row (indicated by ⟵ ). Corresponding variable in the key row known as the leaving (departing) variable.
  - ✓ **'Key Element'** is the number that lies at the junction of the key column and key row.
  - ✓ Constructing Second Tableau.
    - ▪ **New values for the key row** are computed by simply dividing every element of key row by key element.
    - ▪ **New values of the elements in the remaining rows** for the new table can be obtained by performing elementary row operation on all rows so that all elements except the key element in the key column are zero. For each row other than the key row, we use following formula.

      New row number = Number in old rows-{(corresponding number in key row) × (Corresponding fixed ratio)}

      $$\text{Corresponding fixed ratio} = \frac{\text{Old row number in key column}}{\text{Key number}}$$

  Compute $Z_j$ and $C_j – Z_j$ rows. If all the numbers in $C_j – Z_j$ rows are either negative or Zero, an optimum solution has been obtained. *If any of the numbers in $C_j – Z_j$ row is positive repeat the steps as explained above until an optimum solution has been obtained.*

Thus slack variables represent the quantity of a resource not used by a particular solution, and they are necessary to convert the constraint inequalities to equalities.

## MINIMIZATION CASE (PROBLEM) - "TWO PHASES" METHOD

The two phase method, under simplex methodology, is the other method to be studied for minimization problems. Study of the business case and formulation are done as per earlier methodology.

**Standard form**: the constraints are written but while writing the objective function artificial variables are not associated with M. the coefficients are only 1 each. As the name suggests there are two phases to this method. The objective function is divided into two parts.

**Phase I**: the objective function is maximized to remove artificial variables by associating artificial variables with -1. All other variables will have zero as coefficients. The simplex table with optimal solution (maximization) for Phase I doesn't contain artificial variables.

**Phase II**: Putting original values as coefficients for other variables in objective function, objective function is minimized. The optimal solution (minimization) to Phase II gives the optimal solution to the original problem.

**ILLUSTRATION:** Food X1 contains 20 units of vitamin A and 40 units of vitamin B per gramme; Food X2 contains 30 units each of vitamin A and B per gramme. The minimum daily requirements for an individual are 900 units of A and 1200 units of B. How many grammes of each food must be consumed to satisfy daily vitamin requirements at minimum cost? If X1 costs 60 paise per gram and X2 costs 80 paise per gram, find the optimal solution to minimize the costs using simplex method.

**Formulation**

***Decision variables***: Unknown production quantities X1 & X2 are decision variables.

***Objective function***: 60X1 + 80X2 = G (G is the total cost per day which is to be minimized)

***Constraints***:

20X1+30X2 ≥ 900 ………. (1) Vitamin A Constraint (at-least)

40X1+30X2 ≥ 1200……….. (2) Vitamin B Constraint (at-least)

X1 , X2 ≥ 0………………Non-negativity constraint

**Standard form:** The mathematical formulation suitable for simplex method is written in terms of decision variables, surplus variables, artificial variables and their coefficients. While X1 & X2 are decision variables, S1 & S2 are surplus variables introduced to establish equality between LHS & RHS, and A1 & A2 are artificial variables required as per simplex method. A1 & A2 serve as artificial slack variables.

***Objective function***: 60X1 + 80X 2+ 0S1+0 S2+ A1+A2= G

(G is the total cost per day which is to be minimized)

***Constraints***:

20X1 + 30X2- S1+ 0S2+A1+0A2 = 900 ……….(1) Vitamin A Constraint (at-least)

40X1 + 30X2+ 0S1- S2+ 0A1+A2= 1200………(2) Vitamin B Constraint (at-least)

X1, X2, S1, S2, A1, A2 ≥ 0………………Non negativity constraint

***Starting feasible solution***: Put all non-slack variables equal to zero, then

X1=0, X2=0, S1=0, S2=0; A1=900, A2 =1200.

**Structuring of Simplex table**

Put the starting feasible solution in the first simplex table ST1.

**Phase I:**

***Objective function:*** Maximize G* = 0X1 + 0X 2+ 0S1+0 S2- A1 - A2 to drive out A1 & A2

The simplex table ST1 is constructed

**$ST_1$**

| Contributions >>> | | 0 | 0 | 0 | 0 | -1 | -1 | | |
|---|---|---|---|---|---|---|---|---|---|
| C contribution of basic variables | x basic variables | $X_1$ | $X_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | $b_i$ Solution values | θ |
| -1 | $A_1$ | 20 | 30 | -1 | 0 | 1 | 0 | 900 | 45 |
| -1 | $A_2$ | **40** | 30 | 0 | -1 | 0 | 1 | 1200 | 30 |
| | Z | -60 | -60 | 1 | 1 | -1 | -1 | | |
| | Δ = C - Z | 60 | 60 | -1 | -1 | 0 | 0 | | |

The above table doesn't give an optimal solution as all NER values are not non positive. As per the methodology first iteration is performed and next table ST2 is written.

**ST2**.

| Contributions >>> | | 0 | 0 | 0 | 0 | -1 | -1 | | |
|---|---|---|---|---|---|---|---|---|---|
| C contribution of basic variables | x basic variables | $X_1$ | $X_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | $b_i$ Solution values | θ |
| -1 | $A_1$ | 0 | **15** | -1 | 1/2 | 1 | -1/2 | 300 | 20 |
| 0 | $X_1$ | 1 | 3/4 | 0 | -1/40 | 0 | 1/40 | 30 | 40 |
| | Z | 0 | -15 | 1 | -1/2 | -1 | ½ | | |
| | Δ = C - Z | 0 | 15 | -1 | 1/2 | 0 | -3/2 | | |

The above table doesn't give an optimal solution as all NER values are not non positive. As per the methodology first iteration is performed and next table ST3 is written. As A1 & A2 are already driven out the columns of A1 & A2 are dropped.

**ST3.**

| Contributions >>> | | 0 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|
| C contribution of basic variables | x basic variables | $X_1$ | $X_2$ | $S_1$ | $S_2$ | $b_i$ Solution values |
| 0 | $X_2$ | 0 | 1 | -1/15 | 1/30 | 20 |
| 0 | $X_1$ | 1 | 0 | 1/20 | -1/20 | 15 |
| | Z | 0 | 0 | 0 | 0 | |
| | $\Delta = C - Z$ | 0 | 0 | 0 | 0 | |

It can be seen that the solution in ST3 is optimal as all NER values are non-positive.

**Phase II**:

The solution in ST3 is now minimized in Phase II. Objective function is now changed by rewriting the coefficients of variables. The artificial variables are already driven out.

***Objective function:*** Minimize G = 60X1 + 80X 2+ 0S1+0 S2

**ST4.**

| Contributions >>> | | 60 | 80 | 0 | 0 | |
|---|---|---|---|---|---|---|
| C contribution of basic variables | x basic variables | $X_1$ | $X_2$ | $S_1$ | $S_2$ | $b_i$ Solution values |
| 80 | $X_2$ | 0 | 1 | -1/15 | 1/30 | 20 |
| 60 | $X_1$ | 1 | 0 | 1/20 | -1/20 | 15 |
| | Z | 60 | 80 | -7/3 | -1/3 | |
| | $\Delta = C - Z$ | 0 | 0 | 7/3 | 1/3 | |

It can be seen that the solution in ST4 is optimal as all NER values are non-negative. Learners may put the solution values of basic variables in the objective function and find the optimal minimum cost.

***Objective function***: 60X1 + 80X2 = G (G is the total cost per day which is to be minimized). Substituting the values of the decision variables,

60 X 15 + 80 X 20 = 900 + 1600 = 2500.

As the cost contributions are written in paise, the total minimum cost is Rs 25/-

## TRANSPORTATION PROBLEM

## Optimality Test (MODI Method)

**Example 1: Find the initial basic solution for the transportation problem and hence solve it.**

**Destination**

| | | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|---|
| | 1 | 4 | 2 | 7 | 3 | 250 |
| Source | 2 | 3 | 7 | 5 | 8 | 450 |
| | 3 | 9 | 4 | 3 | 1 | 500 |
| Demand | | 200 | 400 | 300 | 300 | |

**The Initial BFS using VAM is**

**Destination**

| | | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|---|
| | 1 | 4 | 2<br>250 | 7 | 3 | 250 |
| Source | 2 | 3<br>200 | 7 | 5<br>250 | 8 | 450 |
| | 3 | 9 | 4<br>150 | 3<br>50 | 1<br>300 | 500 |
| Demand | | 200 | 400 | 300 | 300 | |

Test for optimality using modified distribution method. Compute the values of $U_i$ and $V_j$ for rows and columns respectively by applying the formula for occupied cells.

$C_{ij}+U_i+V_j = 0$

Then, the opportunity cost for each unoccupied cell is calculated using the formula $\overline{C_{ij}} = C_{ij} + U_i + V_j$ and denoted at the left hand bottom corner of each unoccupied cell. The computed valued of $u_i$ and $v_i$ and are shown in table below

**Destination**

| Source | 1 | 2 | 3 | 4 | Supply |
|---|---|---|---|---|---|
| 1 | [4] 5 | [2] 250 | [7] 6 | [3] 4 | 250 $U_1 = 2$ |
| 2 | [3] 200 | [7] 1 | [5] 250 | [8] 5 | 450 $U_2 = -2$ |
| 3 | [9] 8 | [4] 150 | [3] 50 | [1] 300 | 500 $U_3 = 0$ |
| Demand | 200 | 400 | 300 | 300 | |
| | $V_1 = 1$ | $V_2 = 4$ | $V_3 = -3$ | $V_4 = 1$ | |

Calculate the values of Ui and Vj, using the formula for occupied cells. Assume any one of Ui and Vj Value as zero (U3 is taken as 0)

$C_{ij} + U_i + V_j = 0$

$4 + 0 + V_2 = 0,\ \ V_2 = -4$

$5 + V_2 - 3 = 0,\ \ U_2 = -2$

$3 - 2 + V_1 = 0,\ V_1 = -1$

$2 - 4 + U_1 = 0,\ U_1 = 2$

Calculate the values of $\overline{C_{ij}}$, using the formula for unoccupied cells

$\overline{C_{ij}} = C_{ij} + U_i + V_j$

$C_{11} = 4+2 -1 = 5$

$C_{13} = 7+2 -3 = 6$

$C_{14} = 3+2 -1 = 4$

$C_{22} = 7-2 - 4 = 1$

$C_{24} = 8-2 - 1 = 5$

$C_{31} = 9 +0 -1 = 8$

Since all the opportunity cost, $\overline{C_{ij}}$ values are positive the solution is optimum.

Total transportation cost $= (2 \times 25) + (3 \times 200) + (5 \times 250) + (4 \times 150) + (3 \times 50) + (1 \times 300)$

$= 50 + 600 +1250 + 600 + 150 + 300$

= Rs 2,950/-

**Example 2: Obtain an optimal solution for the transportation problem by MODI method given in Table below**

**Destination**

| Source | | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|---|
| | $S_1$ | 19 | 30 | 50 | 10 | 7 |
| | $S_2$ | 70 | 30 | 40 | 60 | 9 |
| | $S_3$ | 40 | 8 | 70 | 20 | 18 |
| | Demand | 5 | 8 | 7 | 14 | |

**Destination**

| Source | | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|---|
| | $S_1$ | 19 | 30 | 50 | 10 | ~~7~~ (9) (9) (40) (40<br>~~2~~ 0 |
| | $S_2$ | 70 | 30 | 40<br>7 | 60<br>2 | ~~9~~ (10) (20) (20) (20)<br>~~2~~ 0 |
| | $S_3$ | 40 | 8<br>8 | 70 | 20 | ~~18~~ (12) (20) (50) --<br>~~10~~ 0 |
| | Demand | ~~5~~ 0 | ~~8~~ 0 | ~~7~~ 0 | ~~14~~ | ~~4~~ ~~2~~ 0 |
| | | (21) | (22) | (10) | (10) | |
| | | (21) | -- | (10) | (10) | |
| | | -- | -- | (10) | (10) | |
| | | -- | -- | (10) | (50)↑ | |

Total transportation cost = $(19 \times 5) + (10 \times 2) + (40 \times 7) + (60 \times 2) + (8 \times 8) + (20 \times 10)$

= $95 + 20 + 280 + 120 + 64 + 200$

= Rs. 779.00

To check for degeneracy, verify the number of allocations, $N = m+n-1$.
In this problem, number of allocation is 6 which is equal $m+n-1$.

$\therefore N = m + n - 1$

$6 = 3 + 4 - 1$

$6 = 6$ therefore degeneracy does not exist.

Test for optimality using MODI method. In Table 6.36 the values of $U_i$ and $V_j$ are calculated by applying the formula $C_{ij} + U_i + V_j = 0$ for occupied cells , and $\overline{C_{ij}} = C_{ij} + U_i + V_j$ for unoccupied cells respectively.

| Source | | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|---|
| | $S_1$ | 19<br>5 | 30<br>32 | 50<br>60 | 10<br>2 | 7 $U_1 = 0$ |
| | $S_2$ | 70<br>1 | 30<br>-18 | 40<br>7 | 60<br>2 | 9$U_2$ = -50 |
| | $S_3$ | 40<br>11 | 8<br>8 | 70<br>70 | 20<br>10 | 18 $U_3 = -10$ |
| | Demand | 5 | 8 | 7 | 14 | |

$V_1 = -19$ $V_2 = 2$ $V_3 = 10$ $V_4 = -10$

Find the values of the dual variables $U_i$ and $V_j$ for occupied cells.

Initially assume $U_i = 0$,

$C_{ij} + U_i + V_j = 0$,

$19 + 0 + V_i = 0, \quad V_1 = -19$

$10 + 0 + V_4 = 0, \quad V_4 = -10$

$60 + U_2 - 10 = 0, \quad U_2 = -50$

$20 + U_3 - 10 = 0, \quad U_3 = -10$

$8 - 10 + V_2 = 0, \quad V_2 = 2$

$40 - 50 + V_3 = 0, \quad V_3 = 10$

Find the values of the opportunity cost, $\overline{C_{ij}}$ for unoccupied cells,

$\overline{C_{ij}} = C_{ij} + U_i + V_j$

$C_{12} = 30 + 0 + 2 = 32$

$C_{13} = 50 + 0 + 10 = 60$

$C_{21} = 70 - 50 - 19 = 1$

$C_{22} = 30 - 50 + 2 = -18$

$C_{31} = 40 - 10 - 19 = 11$

$C_{33} = 70 - 10 + 10 = 70$

In Table the cell (2,2) has the most negative opportunity cost. This negative cost has to be converted to a positive cost without altering the supply and demand value.

Construct a closed loop . Introduce a quantity + q in the most negative cell ($S_2$, $D_2$ ) and a put – q in cell ($S_3$, $D_2$ ) in order to balance the column $D_2$. Now, take a right angle turn and locate an occupied cell in column $D_4$. The occupied cell is ($S_3$, $D_4$) and put a + q in that cell. Now, put a – q in cell ($S_2$, $D_4$ ) to balance the column $D_4$. Join all the cells to have a complete closed path. The closed path is shown in Figure 6.5.

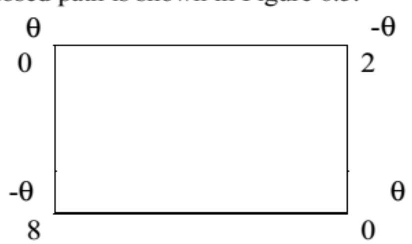
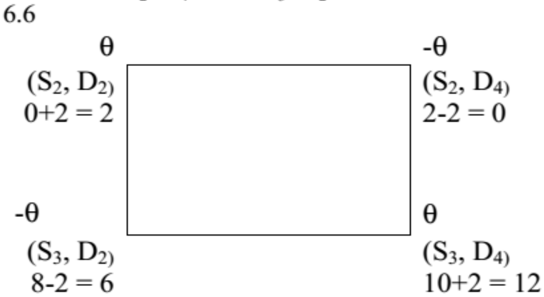



**Figure 6.5: Closed Path**

Now, identify the – q values, which are 2 and 8. Take the minimum value, 2 which is the allocating value. This value is then added to cells ($S_2$, $D_2$ ) and ($S_3$, $D_4$ ) which have '+' signs and subtract from cells ($S_2$, $D_4$ ) and ($S_3$, $D_2$ ) which have '–' signs. The process is shown in Figure 6.6



| | Destination | | | | |
|---|---|---|---|---|---|
| Source | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
| $S_1$ | [19] 5 | [30] 32 | [50] 60 | [10] 2 | 7 |
| $S_2$ | [70] 1 | [30] θ, – 18 | [40] 7 | [60] 2 – θ | 9 |
| $S_3$ | [40] 11 | [8] -θ 8 | [70] 70 | [20] 10 θ | 18 |
| Demand | 5 | 8 | 7 | 14 | |

The table after reallocation is shown in Table below

| | Destination | | | | |
|---|---|---|---|---|---|
| Source | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
| $S_1$ | [19] 5 | [30] | [50] | [10] 2 | 7 |
| $S_2$ | [70] | [30] 2 | [40] 7 | [60] | 9 |
| $S_3$ | [40] | [8] 6 | [70] | [20] 12 | 18 |
| Demand | 5 | 8 | 7 | 14 | |

Now, again check for degeneracy. Here allocation number is 6.
Verify whether number of allocations,

$N = m + n - 1$
$6 = 3 + 4 - 1$
$6 = 6$

therefore degeneracy does not exist.

For occupied cells, $C_{ij} + U_i + V_j = 0$

$19 + 0 + V_1 = 0, \quad V_1 = -19$
$10 + 0 + V_4 = 0, \quad V_4 = -10$
$20 + U_3 - 10 = 0, \quad U_3 = -10$
$8 - 10 + V_2 = 0, \quad V_2 = 2$
$30 + U_2 + 2 = 0, \quad U_2 = -32$
$40 - 50 + V_3 = 0, \quad V_3 = -10$

For unoccupied cells, $\overline{C_{ij}} = C_{ij}+U_i+V_j$

$C_{12} = 30 + 0 + 20 = 50$
$C_{13} = 50 + 0 - 8 = 42$
$C_{21} = 70 - 32 - 19 = 19$
$C_{24} = 60 - 32 - 10 = 18$
$C_{31} = 40 - 10 - 19 = 11$
$C_{33} = 70 - 10 - 8 = 52$

The values of the opportunity cost $\overline{C_{ij}}$ are positive. Hence the optimality is reached. The final allocations are shown in Table

| | | Destination | | | | |
|---|---|---|---|---|---|---|
| | | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
| Source | $S_1$ | 19<br>5 | 30 | 50 | 10<br>2 | 7 $U_1 = 0$ |
| | $S_2$ | 70 | 30<br>2 | 40<br>7 | 60 | 9 $U_2 = -32$ |
| | $S_3$ | 40 | 8<br>6 | 70 | 20<br>12 | 18 $U_3 = -10$ |
| Demand | | 5 | 8 | 7 | 14 | |

$V_1 = -19 \quad V_2 = 2 \quad V_3 = -8 \quad V_4 = -10$

Total transportation cost $= (19 \times 5) + (10 \times 2) + (30 \times 2) + (40 \times 7) + (8 \times 6) + (20 \times 12)$

$= 95 + 20 + 60 + 280 + 48 + 240$

$=$ Rs. 743

**MAXIMIZATION TRANSPORTATION PROBLEM**

In this type of problem, the objective is to maximize the total profit or return. In this case, convert the maximization problem into minimization by subtracting all the unit cost from the highest unit cost given in the table and solve.

**Example 3: A manufacturing company has four plants situated at different locations, all producing the same product. The manufacturing cost varies at each plant due to internal and external factors. The size of each plant varies, and hence the production capacities also vary. The cost and capacities at different locations are given in the following table**:

| Particulars | Plant | | | |
|---|---|---|---|---|
| | A | B | C | D |
| Production cost per unit (Rs.) | 18 | 17 | 15 | 12 |
| Capacity | 150 | 250 | 100 | 70 |

The company has five warehouses. The demands at these warehouses and the transportation costs per unit are given in the Table below. The selling price per unit is Rs. 30/-

| Warehouse | Transportation cost (Rs) — Unit-wise | | | | Demand |
|---|---|---|---|---|---|
| | A | B | C | D | |
| 1 | 6 | 9 | 5 | 3 | 100 |
| 2 | 8 | 10 | 7 | 7 | 200 |
| 3 | 2 | 6 | 3 | 8 | 120 |
| 4 | 11 | 6 | 2 | 9 | 80 |
| 5 | 3 | 4 | 8 | 10 | 70 |

(i) Formulate the problem to maximize profits.
(ii) Determine the solution.
(iii) Find the total profit.

**Solution**:

(i) The objective is to maximize the profits. Formulation of transportation problem as profit matrix table is shown in Table below. The profit values are arrived at as follows.

Profit = Selling Price – Production cost –Transportation cost

**Profit Matrix**

Destination

| | A | B | C | D | Demand |
|---|---|---|---|---|---|
| 1 | 6 | 4 | 10 | 15 | 100 |
| 2 | 4 | 3 | 8 | 11 | 200 |
| 3 | 10 | 7 | 12 | 10 | 120 |
| 4 | 1 | 7 | 13 | 9 | 80 |
| 5 | 9 | 9 | 7 | 8 | 70 |
| **Supply** | 150 | 250 | 100 | 70 | 570 |

Converting the profit matrix to an equivalent loss matrix by subtracting all the profit values from the highest value 13. Subtracting all the values from 13, the loss matrix obtained is shown in the Table below

**Loss Matrix**

Destination

| | A | B | C | D | Demand |
|---|---|---|---|---|---|
| 1 | 9 | 11 | 5 | 0 | 100 |
| 2 | 11 | 12 | 7 | 4 | 200 |
| 3 | 5 | 8 | 3 | 5 | 120 |
| 4 | 14 | 8 | 2 | 6 | 80 |
| 5 | 6 | 6 | 8 | 7 | 70 |
| **Supply** | 150 | 250 | 100 | 70 | 570 |

Now Proceed with VAM.

**Advanced Probability Notes**

## Bayes Theorem

Bayes theorem defines the probability of an event based on the prior knowledge of the conditions related to the event. Let E1, E2,…,En be a set of events associated with a sample space S, where all the events E1, E2,…, En have nonzero probability of occurrence. Let A be any event associated with S, then according to Bayes theorem,

$$P(E_i \mid A) = \frac{P(E_i)\, P(A \mid E_i)}{\sum P(E_k)\, P(A \mid E_k)} \quad \text{for any } k = 1, 2, 3, \ldots, n$$

**OR**

If an event D can occur only in combination with any of the n mutually exclusive and exhaustive events A1, A2, ...... An and if, in an actual observation, D is found to have occurred, then the probability that it was preceded by a particular event Ak is given by

$$P(A_k / D) = \frac{P(A_k).P(D/A_k)}{\sum_{i=1}^{n} P(A_i).P(D/A_i)}$$

**Example 1:** A man is known to speak truth 2 out of 3 times. He throws a die and reports that the number obtained is a four. Find the probability that the number obtained is actually a four.

Solution:

Let A be the event that the man reports that number 4 is obtained.

Let E1 be the event that 4 is obtained and E2 be its complementary event (4 not obtained).

Then, P(E1) = Probability that 4 occurs = 1/6

P(E2) = Probability that 4 does not occurs = 1 – P(E1) = 1 −1/6 = 5/6

Also, P(A|E1) = Probability that man reports 4 and it is actually a 4 = 2/3

P(A|E2) = Probability that man reports 4 and it is not a 4 = 1/3

By using Bayes' theorem, probability that number obtained is actually a four,

$$P(E_1/A) = \frac{P(E_1)\, P(A/E_1)}{P(E_1)\, P(A/E_1) + P(E_2)\, P(A/E_2)} = \frac{(\frac{1}{6} \times \frac{2}{3})}{(\frac{1}{6} \times \frac{2}{3}) + (\frac{5}{6} \times \frac{1}{3})} = 2/7$$

**Example 2:** An Economist believes that during periods of high economic growth, the Indian Rupee appreciates with probability 0.70; in periods of moderate economic growth, it appreciates with probability 0.40; and during periods of low economic growth, the Rupee appreciates with probability 0.20. During any period of time the probability of high economic growth is 0.30; the probability of moderate economic growth is 0.50 and the probability of low economic growth is 0.20. Suppose the Rupee value has been appreciating during the present period. What is the probability that we are experiencing the period of (a) high, (b) moderate, and (c) low, economic growth?

***Solution***: Our partition consists of three events: High economic growth (event H),
Moderate economic growth (event M) and
Low economic growth (event L)

The prior probabilities of these events are: P(H)= 0.30  P(M)= 0.50  P(L)= 0.20

Let A be the event that the rupee appreciates. We have the conditional probabilities

P(A / H)= 0.70  P(A / M)= 0.40  P(A / L)= 0.20

By using the Bayes' theorem we can find out the required probabilities

P(H/A), P(M/A) and P(L/A)

$$P(H/A) = \frac{P(A/H).P(H)}{P(A/H).P(H) + P(A/M).P(M) + P(A/L).P(L)}$$
$$= \frac{(0.70)(0.30)}{(0.70)(0.30) + (0.40)(0.50) + (0.20)(0.20)}$$
$$= 0.467$$

$$P(M/A) = \frac{P(A/M).P(M)}{P(A/H).P(H) + P(A/M).P(M) + P(A/L).P(L)}$$
$$= \frac{(0.40)(0.50)}{(0.70)(0.30) + (0.40)(0.50) + (0.20)(0.20)}$$
$$= 0.444$$

$$P(L/A) = \frac{P(A/L).P(L)}{P(A/H).P(H) + P(A/M).P(M) + P(A/L).P(L)}$$
$$= \frac{(0.20)(0.20)}{(0.70)(0.30) + (0.40)(0.50) + (0.20)(0.20)}$$
$$= 0.089$$

**Example 3**: A manufacturing firm purchases a certain component, for its manufacturing process, from three sub-contractors A, B and C. These supply 60%, 30% and 10% of the firm's requirements, respectively. It is known that 2%, 5% and 8% of the items supplied by the respective suppliers are defective. On a particular day, a normal shipment arrives from each of the three suppliers and the contents get mixed. A component is chosen at random from the day's shipment: (a) What is the probability that it is defective? (b) If this component is found to be defective, what is the probability that it was supplied by (i) A, (ii) B, (iii) C ?

***Solution***: Let A be the event that the item is supplied by A. Similarly, B and C denote the events that the item is supplied by B and C respectively. Further, let D be the event that the item is defective. It is given that: P (A) = 0.6, P(B) = 0.3, P(C) = 0.1,

P (D/A) = 0.02; P (D/B) = 0.05, P (D/C) = 0.08.

(a) We have to find P(D)

From equation (1), we can write

$$P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D)$$
$$= P(A)P(D/A) + P(B)P(D/B) + P(C)P(D/C)$$
$$= 0.6 \times 0.02 + 0.3 \times 0.05 + 0.1 \times 0.08 = 0.035$$

(b) (i) We have to find P(A/D)

$$P(A/D) = \frac{P(A)P(D/A)}{P(D)} = \frac{0.6 \times 0.02}{0.035} = 0.343$$

Similarly, (ii) $P(B/D) = \frac{P(B)P(D/B)}{P(D)} = \frac{0.3 \times 0.05}{0.035} = 0.429$

and (iii) $P(C/D) = \frac{P(C)P(D/C)}{P(D)} = \frac{0.1 \times 0.08}{0.035} = 0.228$

# SILIGURI INSTITUTE OF TECHNOLOGY
## DEPARTMENT OF MASTER OF BUSINESS ADMINISTRATION (MBA)
## Assignment
(For Advanced Learners)
**MBA 2nd SEMESTER'22**

PAPER**: Operations Management** CODE: **MB 204**

**Date of Submission:** 28.04.2022

**MCQ/Very Short Answer Type** [CO 1]

Q1. Which of the following is correct?
- (a) Operations Management is a value addition process.
- (b) Operations Management is conversion of inputs into output.
- (c) Both (a) and (b) are correct.
- (d) Both (a) and (b) are incorrect.

Q2. Which of the following is correct?
- (a) Operations Management is applicable only in manufacturing sector.
- (b) Operations Management is useful in agriculture and manufacturing sector.
- (c) Operation Management is useful in agriculture, manufacturing and services sector.
- (d) None of the above

Q3. Which of the following is correct?
- (a) Productivity is ratio of output to input.
- (b) Productivity is product of output and input.
- (c) Productivity is sum of output and input.
- (d) Productivity is ratio of input to output.

Q4. Which of the following is correct?
- (a) Process focused systems are flexible.
- (b) Process focused systems are used for custom products.
- (c) Process focused systems use general purpose machines.
- (d) All of the above

Q5. Which of the following is correct?
- (a) Product focused systems are used for standard products.
- (b) Product focused systems use special purpose machines.
- (c) Product focused systems have little flexibility.
- (d) All of the above

Q6. Layout of a hospital where a patient receives a number of medical services is
- (a) Product Layout
- (b) Process Layout
- (c) Fixed Position Layout
- (d) Group Technology Layout

Q7. Which of the given set of factors is most appropriate in selecting a plant location?
- (a) Land availability; Labour cost; Proximity to Material; Environmental restrictions
- (b) Land cost; Labour skill available; Capacity requirement; Closeness to market
- (c) Road access, Labour skill, Land cost, Trade unionism
- (d) Sales forecast; Climate; Land cost, Labour availability

Q8. Operations Management is applicable
- (a) Mostly to the service sector
- (b) to services exclusively
- (c) To manufacturing & service sectors
- (d) Mostly to manufacturing sector

Q9. Main objective of assembly Line balancing is to reduce

(a) Total number of operations (b) work element time

` (c) Balance Delay (d) machine load

Q10. Main objective of Production Scheduling is

(a) To meet due dates (b) To measure deviations from planning

(c) To estimate correct requirements (d) none of these

Q11. 'Work Measurement' is done for

(a) To raise productivity (b) determining standard time

(c) Developing an improved method (d) both (a) & (c)

Q12. Expected activity duration calculation in PERT method follows

(a) α (alpha) distribution (b) β (Beta) distribution

(c) Normal distribution (d) Poisson distribution

Q13. Preventive maintenance refers to

(a) System of scheduled & planned maintenance to minimize equipment breakdown (b) Repair work undertaken after the failure of machine or equipment

(c) Maintenance undertaken on the prediction of any fault in the machine

(d) None of these

Q14. What do we call the irreducible minimum time required to complete an activity?

a. Standard time
b. Normal time
c. Crash time
d. Pessimistic time

Q15. _____ refers to assignment of task or work to a facility.

a. Loading
b. Directing
c. Controlling
d. Coordinating

Q16. Some of the common objectives of a good layout are as following

i] Efficient utilization of available space
ii] Minimization of rejections
iii] Economy in material handling
iv] Minimization of production delays

a. i and ii
b. i and iii
c. i, iii and iv
d. All the above

Q17. What are the advantages of Critical Path Analysis?

i] It ensures a through pre-planning
ii] It indicates the optimal start and finish times of each activity of the project
iii] It provides equal weightages to control all activities
iv] It improves task coordination

a. Only i and ii
b. Only i and iii
c. Only ii and iv
d. All i, ii and iv

Q18. Which of the following explain the need for facility location selection?

(a) When the existing business unit has outgrown its original facilities and expansion is not possible.

(b) When a business is newly started.

(c) When the lease expires and the landlord does not renew the lease. (d) All of these.

Q19. Which of the following is the first step in making a correct location choice?
(a) Develop location alternatives
(b) Decide the criteria for evaluating location alternatives
(c) Evaluate the alternatives
(d) Make a decision and select the location

Q20. Which of the following technique emphasizes transportation cost in the determination of facility location?
(a) Location rating factor technique
(b) Transportation technique
(c) Centre-of-gravity technique
(d) Both (b) and (c)

Q21. Transportation cost mainly depends on which of the following factors?
(a) Distance
(b) Weight of merchandise
(c) Time required for transportation
(d) All of the above

Q22. In which of the following site selection techniques, a weightage between '0' to '1' is provided to factors that influence its location decision?
(a) Location rating factor technique
(b) Transportation technique
(c) Centre-of-gravity technique
(d) None of these

Q23. Which of the following does not cause to production delay?
(a) Shortage of space
(b) Long distance movement of materials
(c) Spoiled work
(d) Minimum material handling

Q24. Process layout is also known as ________.
(a) Functional layout
(b) Batch production layout
(c) Straight line layout
(d) Both (a) and (b)

Q25. Which of the following facility layout is best suited for the intermittent type of production, which is a method of manufacturing several different products using the same production line?
(a) Product layout
(b) Process layout
(c) Fixed position layout
(d) Cellular manufacturing layout

Q26. In which of the following layout type, materials are fed into the first machine and finished products come out of the last machine?
(a) Product layout
(b) Process layout
(c) Fixed position layout
(d) Cellular manufacturing layout

Q27. Which of the following is not an advantage of using product layout?
(a) Minimum material handling cost
(b) Minimum inspection requirement
(c) Specialized supervision requirement
(d) None of these

**Short Answer Type** (5 Marks):

1. Define maintenance. Explain its objectives & importance. (CO2)
2. State different functions of maintenance. (CO2)
   State the core maintenance activities. (CO2)
3. What are the signs of poor maintenance? (CO2)
4. What are the causes of breakdown of machines? (CO2)
5. (a) What are the rationales behind preventive maintenance? (CO2)
   (b) Differentiate between Breakdown and Preventive maintenance. (CO2)
6. Explain the concept of Predictive maintenance. (CO2)
7. Explain different Indices of Maintenance Performance Evaluation (CO2)
8. What is TPM? How does it different from TQM? How is TPM measured? (CO2)
9. Briefly explain OEE with its parameters. (CO2)
10. Explain graphically maintenance cost behaviours. (CO2)
11. Assume Machine A is a single product machine and is theoretically capable of producing 1,000 units every hour. In a 16 hour scheduled production day (e.g. 2 x 8 hour shifts) Machine A could theoretically produce 16,000 good units. On a particular day the equipment/process recorded downtime of 2 hours (e.g. equipment breakdown, waiting on materials etc.). Recorded throughput on the equipment/process was 12,632 units and it was found that only 12,000 of the units were deemed good units (i.e. shippable units). Find OEE. (CO3)

13. Explain the objectives & Benefits of PPC. State and explain in brief the phases of PPC. (CO2)
14. Explain the terms Routing, Loading, Dispatching, Expediting (CO2)
15. Differentiate between Production Planning & Control. (CO2)
16. Discuss functions of PPC (CO2)
17. What are the concepts of 'Horizontal' & 'Vertical Loading'? How does Critical ratio (CR) value help to understand whether the Project is ahead/behind/on the schedule? (CO1)
18. Differentiate between: (a) PERT & CPM tools of Networking; (b) Line Layout & Functional Layout (CO2)
19. Define the terms - 'Standard time' and 'Allowance'.(CO1) Calculate the standard time per unit and also production (in units) per shift of 8 hours duration with the following data:
    Observed time per unit = 5 minutes,
    Rating Factor =120%,
    Total allowances = 30% (CO3)
20. (a) What do you mean by 'Cost of Quality'? Give its components with examples. (CO1)
    (b) Write a short note on 'TQM' or on '6-Sigma'. (CO1)

**Long Answer Type** (15 Marks):

1. Real Estate Company has to decide on the location of its new project. It has narrowed down the choice of new project to three locations A, B, C and data in respect of which are given below: (CO3)

| Particulars | Site A | Site B | Site C |
|---|---|---|---|
| Wages & Salary (Rs) | 20000 | 20000 | 20000 |
| Power & Water Supply expenses (Rs) | 20000 | 30000 | 25000 |
| Raw materials & Other Supplies (Rs) | 80000 | 75000 | 60000 |
| Total Initial Investment (Rs) | 200000 | 300000 | 250000 |
| Distribution cost (Rs) | 50000 | 40000 | 60000 |
| Expected Sales per year(Rs) | 225000 | 250000 | 225000 |

Use suitable criterion and advise the company on the best choice.

2. Apply Johnson's rule to find the optimal sequence for processing following seven jobs and calculate the total span & idle time for jobs. (CO3)

| Job | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| M | 9 | 5 | 8 | 3 | 4 | 1 | 7 |
| N | 2 | 4 | 10 | 5 | 6 | 11 | 6 |

3. Construct the Project Network for a Real Estate Construction firm in Gurgaon, Haryana based on the following data (CO3)

| Activity | A | B | C | D | E | F | G | H | I | J | K |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Predecessor | - | - | - | A | A | D | B | C | C | E,F,G | H |
| Optimistic time | 2 | 4 | 5 | 3 | 3 | 2 | 3 | 1 | 2 | 6 | 2 |
| Most likely time | 5 | 19 | 11 | 9 | 6 | 5 | 6 | 4 | 5 | 12 | 5 |
| Pessimistic time | 8 | 28 | 17 | 27 | 15 | 14 | 15 | 7 | 14 | 30 | 8 |

4. The following figures give the number of defectives in 10 samples, each sample containing 1000 items:-

225, 130, 216, 341, 225, 322, 280, 306, 410, 385

Draw the *p*-chart and comment if the process can be regarded in control or not? (CO3)

05. State the major difference between 'Forward' and 'Backward' types of operations scheduling (CO1). From the following data on Processing time (hours) for six jobs in two machines M1 & M2, determine total Makespan and Idle time on machines (if any). Given job order is M1-M2. (CO3)

| Job Number | J1 | J2 | J3 | J4 | J5 | J6 |
|---|---|---|---|---|---|---|
| Processing time in M1 | 5 | 2 | 13 | 10 | 8 | 12 |
| Processing time in M2 | 4 | 3 | 14 | 1 | 9 | 11 |

# SILIGURI INSTITUTE OF TECHNOLOGY
## DEPARTMENT OF MASTER OF BUSINESS ADMINISTRATION (MBA)
## Advanced Study Materials
(For Advanced Learners)
**MBA 2nd SEMESTER'22**

PAPER**: Operations Management** CODE: **MB 204**

## MAKE-OR-BUY DECISION

Make-or-Buy decision (also called the outsourcing decision) is a judgment made by management whether to make a component internally or buy it from the market. While making the decision, both qualitative and quantitative factors must be considered.

Examples of the qualitative factors in make-or-buy decision are:

- Control over quality of the component,
- Reliability of suppliers, and
- Impact of the decision on suppliers and customers, etc.

The quantitative factors are actually the incremental costs resulting from making or buying the component.

A. Incremental production cost per unit,
b. Purchase cost per unit,
c. Production capacity available to manufacture the component, etc.

**Theoretically, a company has choice of three alternatives before starting for a new product:**

1. Purchase the product complete from a contracted manufacturer.
2. Purchase some components and materials, and manufacture and assemble the balance in its own plants.
3. Manufacture the product completely, starting with the extraction of basic raw materials.

In practice, almost no company considers the third alternative. Some companies, who have no manufacturing units, choose the first alternative and obtain a new product completely from another company & sell the product under their trade marks. But in general, most of the companies make certain components of a product and buy others.

### Criteria for Make or Buy Decision:

***A. Companies prefer own manufacturing and buying only raw material or semi-finished parts in the following cases:***

1. Finished product can be made cheaply by the firm than that by the outside suppliers.
2. Finished product only is manufactured by limited number of outside firms, which are unable to meet the demand.
3. The part has an importance for the firm, and requires extremely close quality control.
4. Requires high investment on facilities, which are not available at supplier's plant.
5. Has a demand that is both stable and relatively large.

***B. Companies will usually buy a finished part from an outside supplier when:***

1. They do not have facilities to make it and there are other profitable opportunities for investing company capital.
2. Existing facilities can be used more economically to make other parts.
3. The skill of personnel employed by company is not readily available to manufacture the part.
4. Patent or other legal barriers prevent the company for making the part.
5. Demand for the part is either temporary or seasonal.

### Arguments in favour of (In-house) Making option

1. Purchase price per unit > Cost of manufacturing one unit
2. Higher market potential for the product and relatively stable demand pattern
3. Components/materials are so typical in nature that no suitable & reliable supplier is available or even if exists, too far to reach.
4. Company has adequate production facilities, system capacity, and high caliber skilled labour, and excellent quality control wing to manufacture some raw materials or parts in house.
5. Company policy keeps maintaining its trade or designing secrecy.

### Arguments against (In-house) making while favouring Outsourcing

1. Demand for firm's good(s) is very much fluctuating and unstable.
2. Unit cost of Purchase < Unit cost of making the raw materials/parts in house.
3. Lack of sufficient production capacity, related infrastructure requirements and inadequate skilled human resource.
4. For some typical materials/parts/subassemblies if a vendor has either a patent of making and supplying those items or high reputation to in time supply of items in good quality at least cost.

### Factors considered in Make/Buy decision

There are a number of factors to consider when making this decision, including the following:

- *Cost*. Which alternative presents the lowest total out-of-pocket cost? Businesses tend to include fixed costs when adding up their internal costs, which is incorrect. Only direct costs should be included in the compilation of the internal cost to manufacture a product in-house. This amount should be compared to the quoted price of a supplier.
- *Capacity*. Will the company have sufficient capacity to produce the product in-house? Alternatively, is the supplier reliable enough to be able to produce the goods in sufficient quantities and in a timely manner?
- *Expertise*. Does the company have sufficient expertise to make the goods in-house? In some cases, a business has experienced such a high rate of product failure that it has no choice but to outsource the work to a supplier.
- *Invested funds*. Does the company have enough cash to purchase the equipment needed for in-house production? If the equipment is already on site, could outsourcing the work allow the equipment to be sold, so that the cash can be used elsewhere? This is a major concern for startup companies, which have little excess cash available to invest in facilities.
- *Bottleneck*. Will shifting production to a supplier ease the burden on the company's bottleneck operation? If so, this can be an excellent reason to buy the goods.
- *Drop shipping option*. A supplier may offer to store the goods at its facility and then ship them directly to the company's customers as they place orders. This approach shifts the burden of investing in inventory to the supplier, which can represent a substantial reduction in working capital.
- *Strategic importance*. How important is the product to the corporate strategy? If it is very important, then it could make more sense to manufacture the product, in order to maintain complete control over it. This option is most likely to be taken if the company has proprietary production technology that it does not want to share with a supplier. Conversely, something having little importance can more easily be shifted to a supplier.

## VENDOR RATING

In order to ensure uninterrupted flow of right quality materials to produce customer-conforming quality goods, it is essential to appraise the quality of materials delivered by the contracted vendor(s) periodically by some quantitative measures/indices. Actually vendors having the bulk-orders or supplying critical inputs, fall under the purview of vendor rating framework. Usually vendors are appraised in their performance annually or biannually.

### Purposes Served By Vendor Rating

1. Vendor rating ensures best value for money spent on costly purchases through savings in time & money by identifying best-in-class vendor.
2. Vendor rating by objective comparison among vendors performance devoid of tunnel vision, ascertains capabilities of the vendor in order to appreciate, disqualify or backlist existing vendors objectively.
3. Vendor rating is a tool to promote a healthy competitive spirit among the existing vendors for enhancing future performance.
4. Vendor rating as a feedback mechanism helps to pinpoint the vendor's critical problem areas or weak areas that need to be improved and also require buyer's guidance.

## Vendor rating Methods

- Categorical method
- Weighted-point scheme
- Cost-ratio technique

### (a) Categorical Plan:

The categorical plan is a sample of all vendor rating schemes. It relies heavily on the judgment and experience of the decision maker. The purchaser maintains a list of his suppliers and their products. The vendor performance is reviewed periodically by an evaluation committee comprising of all representatives. Depending upon the performance, the vendor is given a plus point, neutral or minus. The performance trends over a period of time are built up and the vendor with increasing trend of plus point is chosen.

### (b) Weighted Point Plan:

Quality, delivery or service and price are the three most important attributes of a good supplier. Depending upon the importance, a purchaser attaches to a particular attribute he fixes a weightage for it. The weighted point plan technique enables a purchaser to evaluate a supplier on quantitative basis. This plan is more objective than categorical plan.

Vendor performance ranking is done as per the composite ratings.

$$\text{Vendor's Quality rating } (VR_Q) = \frac{\text{Lots rejected}}{\text{Lots received}} \times \text{Weightage } (W_Q)$$

$$\text{Vendor's Delivery rating } (VR_D) = \frac{\text{Delivery on schedule}}{\text{Total no. of deliveries}} \times \text{Weightage } (W_D)$$

$$\text{Vendor Price rating } (VR_P) = \frac{\text{Lowest price paid}}{\text{Price paid by vendor}} \times \text{Weightage } (W_P)$$

Total Composite Rating for any Vendor = $VR_{TOTAL} = VR_Q \times W_Q + VR_D \times W_D + VR_P \times W_P$

### (c) Cost Ratio Method:

This method relates to identifiable purchasing and receiving costs to the value of shipment received from respective suppliers. The higher the ratio of costs to shipments, the lower the rating applied to the supplier: Quality, delivery, service and price are the usual categories to which costs are allocated, after subdividing each factor into various elements. The respective cost ratios are suitably combined with the vendors' quoted price, to determine the net cost. Here, the vendor performance is reviewed periodically by an evaluation committee comprising of representatives from all departments involved with purchasing.

**Exercise 1:** From the information furnished below, select the best vendor after conducting rating analysis. The company has assigned weightages for Quality: 50%, for Delivery: 25%, for Price: 15% and for suggestion response: 10%. The following table provides various performance data of the three vendors namely – A, B and C as given below:-

| Particulars of Vendor | A | B | C |
|---|---|---|---|
| Quantity received | 1200 | 1500 | 1350 |
| Quantity accepted | 1100 | 1400 | 1050 |
| Basic unit price (Rs) | 6.00 | 5.80 | 6.20 |
| Committed delivery period | 4 weeks | 3 weeks | 4 weeks |
| Actual delivery | 4.2 weeks | 2.9 weeks | 4.5 weeks |
| Suggestions made | 2 | 4 | 3 |

**Solution:**

| Particulars of Vendor | A | B | C |
|---|---|---|---|
| Acceptance (%) =[Quantity Accepted/Quantity Received]x100 Quality Rating = Acceptance x Weightage | (1100/1200)x100 = 91.7 91.7x0.50=45.85 | (1400/1500)x100 = 93.3 93.3x0.50=46.65 | (1050/1350)x100 = 77.8 77.8 x 0.50=38.90 |
| Price ratio(%)=lowest price/price paid Price Rating=Price ratio x Weightage | [5.80/6.00]x100 = 96.67 96.67x0.15=14.50 | [5.80/5.80]x100 = 100 100 x 0.15=15 | [5.80/6.20]x100 = 93.55 93.55x0.15=14.03 |
| Delivery ratio=Promised/Actual delivery Delivery Rating=Delivery Ratio x Weightage | [4.0/4.2] x 100 = 95.2 95.2 x 0.25=23.8 | [3.0/2.9] x 100 = 103.4 103.4 x 0.25=25.8 | [4.0/4.5] x 100 = 88.9 88.9 x 0.25=22.2 |
| Response to Suggestion (%) | [2/9] x 100 =22.2 | [4/9] x 100 =44.4 | [3/9] x 100=33.3 |
| Suggestion Rating= Suggestion(%)x Weight | 22.2 x 0.10 = 2.22 | 44.4 x 0.10 =4.44 | 33.3 x 0.10 = 3.33 |
| **Total Rating** | **86.37** | **91.89** | **78.46** |
| **Rank** | **II** | **I** | **III** |

So the best performing Vendor is Vendor B.

**Example 2:** A garment manufacturer for its vendor rating exercise assigns 40%, 35% and 25% weightages to vendor performance factors quality, price and delivery respectively. Conduct the vendor rating study from the following data:

| Supplier | Inspection results | | Price analysis | | | Delivery analysis |
|---|---|---|---|---|---|---|
| | Received lots | Accepted lots | Basic price | Discount | Transport charge | Delivery missed |
| P | 30 | 27 | 10.00 | 5% | 30% | 15% |
| Q | 30 | 28 | 12.50 | 10% | 20% | 10% |
| R | 12 | 12 | 15.00 | 20% | 25% | Nil |

Solution:

| Rating assessment of Vendor | P | Q | R |
|---|---|---|---|
| Acceptance(%)=[Lot Accepted/Lot Received]x100 Quality Rating = Acceptance x Weightage | (27/30) x 100= 90 91.7 x 0.40=36 | (28/30) x 100= 93.3 93.3 x 0.40=37 | (12/12) x 100 = 100 100 x 0.40=40 |
| Discounted price: Base Price(1-Discount) Add: Transport Charge Actual Price paid Price Rating = Price ratio x Weightage Where Price Ratio = Minimum of Actual Price paid / Respective Vendors Actual Price paid | 9.50 0.30 9.80 [9.80/9.80] x 0.35 = 35 | 11.25 0.20 11.45 [9.80/11.45]x0.35 = 30 | 12.00 0.25 12.25 [9.80/12.25]x0.35 = 29 |
| % Delivery kept=[100-missed%] Delivery Rating=% Delivery Kept x Weightage | 85% 85 x 0.25= 21 | 90% 90 x 0.25 = 23 | 100% 100 x 0.25 = 25 |
| **Total Rating** | **92** | **90** | **94** |
| **Rank** | **II** | **III** | **I** |

## Overall Equipment Effectiveness [OEE]

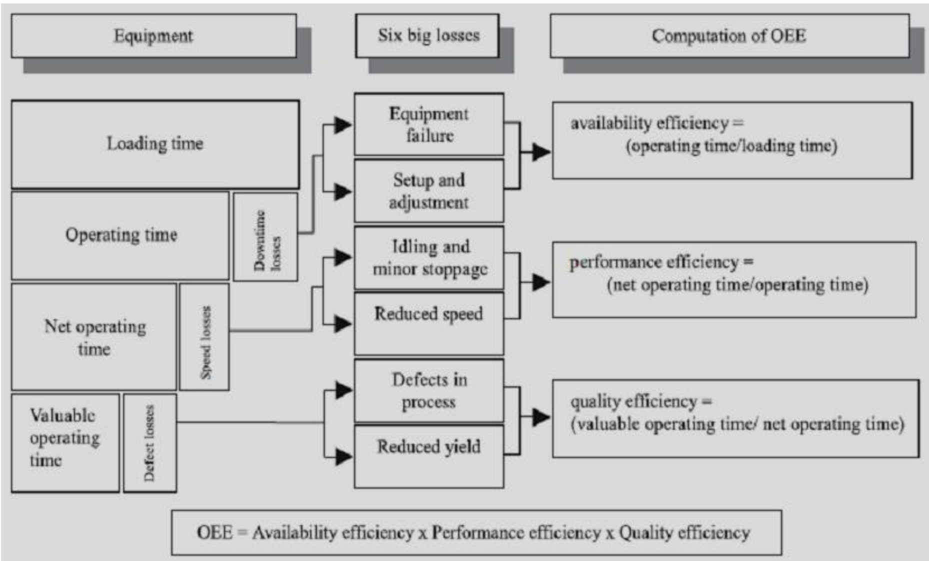
- A method of measuring how well we use our machines and processes to convert raw materials into finished saleable goods.
- A quantitative metric for measuring productivity of individual production equipment in a factory.
- Being a part of total productive performance (TPM) it is regarded as a measurement tool under TPM and at aimed identifying production losses related to equipment.
- Basically OEE is about effectiveness as it is the rate between what a machine theoretically could produce and what it actually did.

Every manufacturing unit strives to accomplish overall equipment effectiveness by eliminating six big losses –

1) Equipment failure
2) Set-up and Adjustment
3) Idling and minor stoppage
4) Reduced speed
5) Defect in process / rework
6) Reduced yield

**Computation of OEE**

OEE = Availability × Performance × Quality




**Example:** Assume Machine A is a single product machine and is theoretically capable of producing 1,000 units every hour. In a 16 hour scheduled production day (e.g. 2 x 8 hour shifts) Machine A could theoretically produce 16,000 good units. On a particular day the equipment/process recorded downtime of 2 hours (e.g. equipment breakdown, waiting on materials etc.). Record throughput on the equipment/process as 12,632 units and it was found that only 12,000 of the units were deemed good units (i.e. shippable units).

**Solution:**

Scheduled Production Time = 16 Hrs (2 x 8Hr Shift)

Downtime (planned & unplanned) = 2 Hrs

Available Time (Uptime) = 16 Hrs - 2 Hrs = 14 Hrs

Available Time / Scheduled Time = 14Hrs/16Hrs = 88% it **is Availability**

Total product produced (Throughput) in Available Time = 12,632 units

Theoretical time to produce 12,632 units = 12,632/1,000 = 12.63 Hrs

Theoretical Time / Available Time = 12.63Hrs / 14Hrs = 90% it is **Performance**

The 632 are accounted as scrapped and/or rework cycles to make shippable.

Performance Time = 12.63 Hrs

Good Units (expressed in time) = 12,000 units / 1,000 per Hour = 12 Hrs

Good Units (expressed in Time) / Performance Time = 12Hrs / 12.63 Hrs

= 95% it **is Quality**

Availability x Performance x Quality = 88% x 90% x 95% = 75% **OEE**

**SILIGURI INSTITUTE OF TECHNOLOGY**
**DEPARTMENT OF MASTER OF BUSINESS ADMINISTRATION (MBA)**
**Participation in Technical Event: Business Baazigarh 2K22**
**(Innovative Business Idea/Plan Presentation Contest)**
(For Advanced Learners)

SILIGURI INSTITUTE OF TECHNOLOGY

TECHNOVISION 20K22

**Report of "BUSINESS BAAZIGARH 2K22"**

Date: 29/04/22

Time: 11 AM -3 PM

Venue: Room No: 203, Department of Business Administration (MBA), SIT

| | |
|---|---|
| Brief Introduction | A new and a creative business idea is a key for healthy venture. Business Baazigarh competition is an opportunity for the students aspiring to be an entrepreneur to unfold their Entrepreneurial Vision and Skill. It is a team based event with minimum two members and maximum four members in a team. Participants need to make a detailed business plan for any innovative product/Service with a Capital of **Rs. 1 crore (Maximum).** All registered team will have to give a Power point presentation on their business plan. The Business plan will be evaluated on the following parameters: Innovativeness, Market feasibility, financial viability, team work, overall presentation of business idea. |
| Objective | To provide a meaningful learning experience for students & to **stimulate entrepreneurship** among students. |
| Outcomes | • Experience creating a business model and writing a business plan<br>• Familiarity with primary / secondary market research and the process of conducting feasibility studies<br>• experience working as part of a team<br>• opportunity to sharpen organizational and communication skill |
| Name of the Judge | Mr. Jyoti Prakash Basu Dhar, Chartered Accountant, Siliguri |
| Total No. of Participants | 25 |
| Committee members | **Faculty Coordinator** : Santana Guha, Asst. Professor<br>**Student members :** Nidhi Bharati, Anik Ghosh Chowdhury, Rikita Chettri & Anurag Sharma |
| Photo gallery | Paste 4-6 photo in a page (max) with captions |

**SNAPSHOTS of Business Baazigarh 2K22_SIT: Tech-Management Fest 2022**



**The Audience**


**Questioning Round by Judges to Presenting Team**


**Team Presentation Going On**


**Team Presentation Going On**

# OPTIMIZATION PROBLEMS in Economics
## [Managerial Economics (Micro): MB 101]

**Q1. Profit maximizing output when Revenue & Cost functions given**

In a perfectly competitive market, the total revenue and total cost of a firm are given by $R = 20q$ and $C = q^2 + 4q + 20$.

Find profit maximizing output and maximum profit.

**Solution**

By definition profit ($\pi$) is the difference between total revenue (R) and total cost (C)

$\therefore \pi = R - C$ or $\pi = 20q - q^2 - 4q - 20$

Since $\pi = f(q)$, the maximum of profit requires

$$\frac{d\pi}{dq} = 0 \text{ and } \frac{d^2\pi}{dq^2} < 0.$$

Now $\frac{d\pi}{dq} = 20 - 2q - 4 = 0$ or $2q = 16$ or $q = 8$

Again $\frac{d^2\pi}{dq^2} = -2 < 0$

Since the second order derivative is negative, $q = 8$ will maximize profit of the firm. The maximum profit is obtained by substituting $q = 8$ in the profit function.

$\therefore$ Maximum profit $= 20 \times 8 - (8)^2 - 4 \times 8 - 20 = 44$

**Q2. Profit maximizing output & Point elasticity when Revenue & Cost functions given**

A monopolist has the following total revenue (R) and total cost (C) functions $R = 30q - q^2$, $C = q^3 - 15q^2 + 10q + 100$

Find

a. Profit maximizing output

b. Maximum Profit

c. Equilibrium price

d. Point elasticity of demand at equilibrium level of output.

**Solution**

a. The profit function $\pi$ is given by

$$\pi = R - C = 30q - q^2 - q^3 + 15q^2 - 10q - 100$$

or $$\pi = -q^3 + 14q^2 + 20q - 100 \qquad \text{... (1)}$$

For profit maximization, $\frac{d\pi}{dq} = 0$ and $\frac{d^2\pi}{dq^2} < 0$

$\frac{d\pi}{dq} = 0$ gives $-3q^2 + 28q + 20 = 0$

or $$3q^2 - 28q - 20 = 0$$

$\therefore$ $$q = \frac{28 \pm \sqrt{(28)^2 - 4 \times 3 \times (-20)}}{2 \times 3}$$

$$= \frac{28 \pm \sqrt{1024}}{6} = \frac{28 \pm 32}{6} = 10 \text{ or } -\frac{2}{3}$$

---

Now $\frac{d^2\pi}{dq^2} = -6q + 28$

When $q = 10$, $\frac{d^2\pi}{dq^2} = -32 < 0$

When $q = -\frac{2}{3}$, $\frac{d^2\pi}{dq^2} = 32 > 0$

$\therefore$ the profit maximizing output is $q = 10$.

b. Substituting $q = 10$ in profit function we get maximum profit $\pi = -(10)^3 + 14(10)^2 + 20 \times 10 - 100 = 500$

c. The price equation or average revenue function is obtain as $\frac{R}{q}$.

$$\text{Price} = \frac{30q - q^2}{q} = 30 - q$$

For equilibrium output $q = 10$, Price = 30–10=20

d. Point Elasticity of demand is define as $\eta = \frac{AR}{AR - MR}$

Now $MR = 30 - 2q = 30 - 2 \times 10 = 10$

and AR = Price = 20 $\quad \therefore \eta = \frac{20}{20-10} = 2$

**Q3. Profit maximizing output & impact of subsidy when Demand & Cost Functions are given**

A firm has the total cost (C) function $C = 7q^2 + 5q + 120$ and demand function $P = 180 - 0.5q$ and also a subsidy of Rs. 5/- per unit of output is paid by the government. Find

a. Profit maximizing output and price

b. Maximum profit

c. Impact of subsidy on equilibrium output and prices.

**Solution**

a. When q units of output is produced, the total cost of subsidy will be 5q. Likewise total revenue is given by

$$R = p.q = (180 - 0.5q)q = 180q - 0.5q^2$$

So profit with subsidy is defined as

$$\pi = TR - TC + \text{ subsidy}$$

$$\therefore \quad \pi = 180q - 0.5q^2 - 7q^2 - 5q - 120 + 5q$$

$$\text{or} \quad \pi = 180q - 0.5q^2 - 7q^2 - 120 \qquad \text{... (1)}$$

$\therefore$ Profit maximization requires that

$$\frac{d\pi}{dq} = 0 \text{ and } \frac{d^2\pi}{dq^2} < 0$$

Now $\frac{d\pi}{dq} = 180 - 2(0.5)q - 14q = 0$

or $180 - q - 14q = 0$ or $15q = 180$ $\quad \therefore q = 12$

Now $\frac{d^2\pi}{dq^2} = 0 - 1 - 14 = -15 < 0$

$\therefore$ With q = 12, the profit will be maximum. The profit maximizing price is obtained by substituting q = 12 in the price equation.

$$P = 180 - 0.5q = 180-(0.5) \times 12 = 180-6=174$$

b. Maximum profit is obtained by putting q = 12 in (1)

$$\therefore \quad \pi=180 \times 12-0.5(12)^2-7(12)^2-120=960$$

c. In order to study the impact of subsidy on equilibrium price and quantity, we are to find out equilibrium values without subsidy. So profit without subsidy $(\pi^*)$ is given by

$$\pi^* = 180q-0.5q^2-7q^2-5q-120$$

Now $\frac{d\pi^*}{dq}=0$ gives $180-2(0.5)q-14q-5=0$

or $175-q=-14q=0$ or $15q=175 \therefore q=11.67$

$$\frac{d^2\pi^*}{dq^2}=-15<0$$

So profit maximizing output without subsidy, q = 11.67.

Substituting q = 11.67 in the price equation, we have

$$P=180-0.5q=180-(0.5)(11.67)$$

$$=180-5.83=174.17$$

So the equilibrium profit without subsidy.

$$\pi^*=180(11.67)-0.5(11.67)^2-7(11.67)^2-120=900$$

Thus equilibrium price, output and profit with and without subsidy indicate that output increases, price falls and also profit increases as a result of provision of subsidy to the firm.

**Q4. Find the maximum profits 'π' for a firm, given total revenue $R = 4000Q - 33Q^2$ and the total cost function $C = 2Q^3 - 3Q^2 + 400Q + 5000$, assuming $Q > 0$.**

Profit function: $\pi = R - C$

$$\Pi = (4000Q - 33Q^2) - (2Q^3 - 3Q^2 + 400Q + 5000)$$

$$= -2Q^3 - 30Q^2 + 3600Q - 5000$$

Taking the first derivative, set it equal to zero, and solving this for Q we get critical points.

$\pi' = -6Q^2 - 60Q + 3600 = 0$

$= -6(Q^2 + 10Q + 600) = 0$

$= -6(Q + 30)(Q - 20) = 0$

$Q = -30 \quad Q = 20$ critical points

Taking the second derivative we get

$\pi'' = -12Q - 60$

$\pi''(20) = -12(20) - 60 = -300 < 0$

[Taking Q = 20 ignoring negative value]

The function is concave and relative maximum. It means profit is maximum at Q = 20

When Q=20 the profit п is

$п(20) = 2(20)^3 - 30(20)^2 + 3600(20) - 5000 = \underline{39000}$

**Q5: From the total cost function TC = $Q^3$– $5Q^2$ + 60Q find (1) the average cost AC function (2) the critical value at which AC is minimized and (3) the minimum average cost.**

$TC = Q^3 - 5Q^2 + 60Q$

(1) $AC = \frac{TC}{Q} = \frac{Q^3 - 5Q^2 + 60Q}{Q} = \underline{\underline{Q^2 - 5Q + 60}}$

(2) AC is minimum at

$AC' = 2Q - 5 = 0, \ \therefore Q = \underline{\underline{2.5}}$

$AC'' = 2 > 0$ convex, relative minimum

(3) $AC(2.5) = (2.5)^2 - 5(2.5) + 60 = \underline{\underline{53.75}}$

**Q6: Given the total revenue R = 15Q – $Q^2$ find the Average Revenue (AR), Marginal Revenue (MR) and the level of output Q that maximizes Total Revenue (TR).**

$TR = R = 15Q - Q^2$

$AR = \frac{TR}{Q} = \frac{15Q - Q^2}{Q} = \underline{\underline{15 - Q}}$

$MR = TR' = \frac{dR}{dQ} = 15 - 2Q$

Setting $TR' = MR = 0$ we get $15 - 2Q = 0$

$\therefore 15 = 2Q$ and $Q = \underline{\underline{7.5}}$ critical value.

$TR'' = -2 < 0$ concave, relative maximum.

TR when Q = 7.5 is

$15(7.5) - (7.5)^2$

$= 112.5 - 56.25 = 56.25$

**The output that maximize TR =7.5; AR when Q=7.5 is 15-7.5 = 7.5; MR when Q =7.5 is 15-2(7.5) =15-15 = 0**

**Ex. 7: A firm has the following total cost, C and demand function Q**

**$C = 1/3\ Q^3 - 7Q^2 + 111\ Q + 50$ and $P = 100 - Q$**

**(1) Write down the total revenue function R in terms of Q**

**(2) Formulate the total profit function**

Solution:

(1) Price function $P = 100 - Q$

Revenue = P x Q

The revenue function is $R = PQ = 100Q - Q^2$

(2) Profit function is $\Pi = TR - TC$ functions

$= (100Q - Q^2) - (1/3\ Q^3 - 7Q^2 + 111Q + 50)$

$\Pi = -1/3\ Q^3 + 6Q^2 - Q - 50$

**Ex 8: From the total cost function TC = Q3– 5Q2 + 60Q find (1) the average cost AC function (2) the critical value at which AC is minimized and (3) the minimum average cost.**

$TC = Q^3 - 5Q^2 + 60Q$

(1) $AC = \frac{TC}{Q} = \frac{Q^3 - 5Q^2 + 60Q}{Q} = Q^2 - 5Q + 60$

(2) AC is minimum at

$AC' = 2Q - 5 = 0, \therefore Q = 2.5$

$AC'' = 2 > 0$ convex, relative minimum

(3) $AC(2.5) = (2.5)^2 - 5(2.5) + 60 = 53.75$

**Ex 9: Given the Total Cost Function $C = 1/3Q^3 - 3Q^2 + 9Q$, find Q when Average Cost is minimum. Find also the Marginal Cost at the level of Q.**

**Solution**

$C = 1/3\ Q^3 - 3Q^2 + 9Q$

$AC = \frac{C}{Q} = \frac{1/3Q^3 - 3Q^2 + 9Q}{Q} = 1/3Q^2 - 3Q + 9$

when AC is minimum

i) First derivative of AC = 0

ii) Second derivative of AC > 0

$d/dQ\ (AC) = \frac{d}{dQ}(1/3Q^2 - 3Q + 9)$

$= 2/3\ Q - 3$

When AC is minimum d/dx(AC) = 0

$2/3\ Q - 3 = 0$

$2/3\ Q = 3$

$Q = 3\ (3/2)$

$Q = 9/2$

when AC is minimum $\frac{d^2}{dQ^2}(AC) > 0$

$\frac{d^2}{dQ^2}(AC) = \frac{d}{dQ}(2/3Q - 3)$

$= 2/3 > 0$

$MC = \frac{d}{dQ}(C) = \frac{d}{dQ}(1/3Q^3 - 3Q^2 + 9Q)$

$MC = Q^2 - 6Q + 9$

Putting Q = 9/2 in MC equation we get MC at Q = 9/2 is = 9/4